Modifying instanton sums in QCD

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Motivation: $U(1)_A$ Problem

In QCD, we have chiral symmetry breaking,

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V,$$

with $N_f^2 - 1$ Nambu-Goldstone (NG) bosons ($N_f = 2$ or 3).

What about $U(1)_A$?

$$\overline{\psi}_R \psi_L \sim \Lambda^3 \exp(\mathrm{i}\eta').$$

Actually, η' is quite heavy:

$$m_{\eta'} = 958\mathrm{MeV}.$$

A resolution of $U(1)_A$: Instantons

A standard answer to $U(1)_A$ would be

"$U(1)_A$ symmetry does not exist because of instantons".

Under the $U(1)_A$ transformation, $\psi \to \exp(\mathrm{i}\alpha\gamma_5)\psi$,

$$\mathcal{D}\overline{\psi}\mathcal{D}\psi \to \mathcal{D}\overline{\psi}\mathcal{D}\psi \exp\Bigl(2\mathrm{i}N_f\alpha \underbrace{\frac{1}{8\pi^2}\int \mathrm{tr}(F\wedge F)}_{Q_{\mathrm{top}}}\Bigr).$$

Instanton configuration has nonzero Q_{top}, so $U(1)_A$ is not symmetry. ('t Hooft, '76)

Mass of η' in large-N

Take large-N limit with fixed $\lambda = g^2 N$. Instanton amplitude is

$$\exp\left(-\frac{8\pi^2}{g^2}\right) = \exp\left(-\frac{8\pi^2}{\lambda} N\right).$$

Thus, origin of $U(1)_A$ breaking is exponentially small, which implies

$$m_{\eta'} \sim O(\exp(-N\#)).$$

This is quite small.

$U(1)_A$ without instantons

Interestingly, $m_{\eta'}$ doesn't require non-zero topological sectors at all! (Witten, '79; Veneziano, '79)

Just the $1/N$ counting of Feynman diagrams show that

$$m_{\eta'}^2 \propto \frac{\chi_{\rm top}}{f_\pi^2} \sim \frac{\Lambda^2}{N}.$$

It is suppressed in $1/N$, but not exponentially suppressed.

Two explanations sound like very different.

Question

Do we really need instantons?

Instanton sums and locality

Instantons and θ vacua

$SU(N)$ gauge field can be classified by instanton numbers

$$Q_{\mathrm{top}} = \nu \in \mathbb{Z}.$$

Partition function with fixed topology:

$$Z_\nu = \int \mathcal{D}[\delta a] \exp\left(-\frac{1}{g_{\mathrm{YM}}^2}\int \mathrm{tr}[F(a_\nu+\delta a)\wedge \star F(a_\nu+\delta a)]\right).$$

Here,

- a_ν: an arbitrary reference gauge field with $Q_{\mathrm{top}} = \nu$.
- $\mathcal{D}[\delta a]$: Path integral over $\mathfrak{su}(N)$-valued one-forms.

We define the theta vacua by

$$Z(\theta_{\mathrm{YM}}) = \sum_{\nu\in\mathbb{Z}} \mathrm{e}^{-\mathrm{i}\,\nu\,\theta_{\mathrm{YM}}} Z_\nu.$$

Locality and instantons

Why do we have to sum up instanton sectors?

Locality of QFT requires that

$$Z_{M_1 \sqcup M_2} = Z_{M_1} Z_{M_2}.$$

However, the partition function on a fixed topological sector doesn't satisfy this requirement:

$$Z_{\nu,\, M_1 \sqcup M_2} = \sum_{\nu' \in \mathbb{Z}} Z_{\nu-\nu',\, M_1} Z_{\nu',\, M_2}$$

Thus, θ-vacua seem to be the unique way to define YM as local and unitary QFT. (Belavin, Polyakov, Schwarz, Tyupkin, '75; Callan, Dashen, Gross, '76)

Now, I can make my naive question more rigorous.

In order to understand the role of instanton sectors, we want to compare theories with different instanton sums.

Question

Can we modify the instanton sum within local, unitary QFT?

Luckily, this has been studied in the string context (Pantev, E. Sharpe, '05; Seiberg, '10), and the answer is YES.

Question

How do the modified and unmodified theories behave differently?

Modified instanton sums in local QFT

Modified instanton sum: First try

Let's try to restrict instanton sectors. Recalling that

$$\int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \exp(\mathrm{i}\nu\theta) = \delta_{\nu,0},$$

one may consider

$$Z_0 = \int \frac{\mathrm{d}\theta}{2\pi} Z(\theta).$$

As θ is not a local field, this does not satisfy locality.

This does not define a local QFT, but close!
Moreover, this formula is useful for lattice QCD in fixed topology (S. Aoki, Fukaya, S. Hashimoto, Onogi, '07).

More on the first try

What is really wrong in the formula?

Why can't we regard θ as a zero-mode of a scalar field χ (axion)?

Assume χ is very massive, whose EOM is approximately given as

$$\partial_\mu \chi = 0.$$

This means that χ is locally a constant, but doesn't have to be globally constant unlike θ.

Modified instanton sum: Coupling a BF theory

We introduce a level-p BF (topological) theory,

$$S = \mathrm{i}\frac{p}{2\pi}\int \chi \wedge \mathrm{d}c^{(3)},$$

where

- χ: 2π-periodic scalar field,
- $c^{(3)}$: 3-form $U(1)$ gauge field.

Note: They are not propagating.

We now introduce a topological coupling to YM,

$$\mathrm{i}\frac{1}{8\pi^2}\int \chi \wedge \mathrm{tr}[F \wedge F].$$

This restricts instanton sectors to $\nu \in p\mathbb{Z}$. (Seiberg, '10)

Our full Lagrangian is

$$S_{\mathrm{gYM}} = \frac{1}{2g^2}\int \mathrm{tr}[F(a)\wedge \star F(a)] + \frac{\mathrm{i}\theta_{\mathrm{YM}}}{8\pi^2}\int \mathrm{tr}[F(a)\wedge F(a)]$$
$$+\mathrm{i}\int \chi \wedge \left(\frac{1}{8\pi^2}\mathrm{tr}[F(a)\wedge F(a)] - \frac{p}{2\pi}\mathrm{d}c^{(3)}\right) + \frac{\mathrm{i}\hat{\theta}}{2\pi}\int \mathrm{d}c^{(3)}.$$

Two ways to understand this modified theory:

- Integration over $c^{(3)} \Rightarrow \chi = \frac{2\pi}{p}\ell$ with $\ell = 0, 1, \dots, p-1$.
- Integration over $\chi \Rightarrow \frac{1}{8\pi^2}\mathrm{tr}[F^2] = \frac{p}{2\pi}\mathrm{d}c^{(3)}$, i.e. $\nu \in p\mathbb{Z}$.

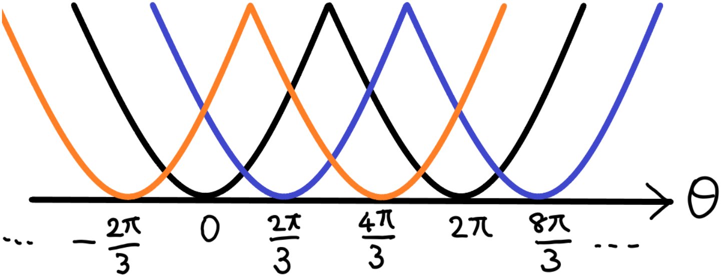
Comments:

- When $p = 1$, this is the same with usual YM.
- When $p > 1$, the instanton sum is modified, without additional local propagating modes.

Dynamical aspects of the YM theory with modified instanton sums

Vacuum structure with p-instanton sums

Let us discuss the dynamical consequence of χ and $c^{(3)}$ with $p > 1$.



This is the θ dependence of $E_{\mathrm{ground\,state}}(\theta)$ (in the large-N limit).

The vacuum periodicity is now $2\pi/p$ (with $p = 3$ in Fig.).

The vacua with different colors have different values of $\chi = 2\pi\ell/p$.

Comments

Any local observables are not affected by the modified instanton sum.
Especially, $\chi_{\rm top}$ is independent of p.
$\Rightarrow$ We have the same mass of η' even for $p=1000$.

Still, the global structure of vacuum is affected by χ and $c^{(3)}$.
The ground-state energy behaves as

$$E(\theta)=\min_{k\in\mathbb{Z}}\frac{\chi_{\rm top}}{2}\left(\theta-\frac{2\pi k}{p}\right)^2.$$

(Y.T., M. Ünsal, 1912.01033)

Note: Both of these comments can be explicitly derived by the large-N counting (generalizing Witten '79, '98).

Derivation by global symmetry

The results can be derived only by using the symmetry.
Global symmetries of this theory are

- $\mathbb{Z}_N^{[1]}$: $W^{(1)}(M_1) = \mathrm{tr}(\mathcal{P}\exp(\mathrm{i}\int_{M_1} a)) \to \mathrm{e}^{2\pi\mathrm{i}/N} W^{(1)}(M_1)$,
- $\mathbb{Z}_p^{[3]}$: $W^{(3)}(M_3) = \exp(\mathrm{i}\int_{M_3} c^{(3)}) \to \mathrm{e}^{2\pi\mathrm{i}/p} W^{(3)}(M_3)$.

We can gauge this symmetry in a consistent manner,
if their corresponding gauge fields $B^{(2)}$ and $D^{(4)}$ satisfies

$$pD^{(4)} = \mathrm{d}D^{(3)} + \frac{N}{4\pi} B^{(2)} \wedge B^{(2)}.$$

(Y.T., M. Ünsal, 1912.01033)

We can obtain the generalized mixed anomaly as

$$Z_{\mathrm{gYM}}(\theta_{\mathrm{YM}} + 2\pi k/p) = \exp\left(\mathrm{i}k \int D^{(4)}\right) Z_{\mathrm{gYM}}(\theta_{\mathrm{YM}})$$

$\Rightarrow$ We need to encounter a phase transition to match this anomaly when dialing θ_{YM} from 0 to $2\pi/p$. (Y. Kikuchi, Y.T., '17)

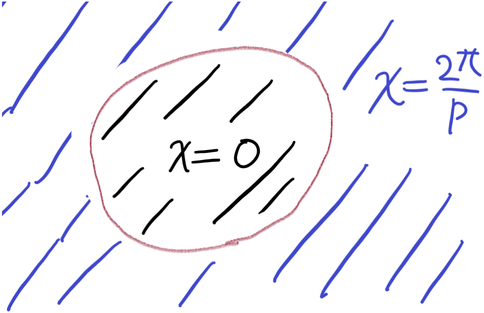
Moreover, only if $k = p\,n$, this anomaly reduces to a mixed anomaly with the 1-form center symmetry (given in Gaiotto, et al. '17).

$\Rightarrow$ When $k \notin p\mathbb{Z}$, the phase transition should be accompanied by the nontrivial jump of χ.

(Y.T., M. Ünsal, 1912.01033)

Restriction of vacuum decay

We consider a higher-energy state (false vacuum) with $\chi = 2\pi/p$ at $\theta = 0$.
Does the false-vacuum decay occur by the nucleation process?



Answer is NO!

EOM requires that χ is locally a constant.
When nucleation occurs, χ jumps from $2\pi/p$ to 0, which violates the requirement by EOM.

$\Rightarrow$ Nucleation of the true vacuum is prohibited.

From the symmetry viewpoint, the wall configuration for this process is charged under the 3-form symmetry $\mathbb{Z}_p^{[3]}$.

Such a configuration is possible if and only if we insert the external operator $W^{(3)}(M_3)$.

Beyond superselection rule (Universes)

States $|\chi = 2\pi/p\rangle$ and $|\chi = 0\rangle$ are distinguished by superselection rule.
They satisfy the stronger selection rule as follows.

We propose to call two states $|\Phi_1\rangle, |\Phi_2\rangle \in \mathcal{H}$ as different universes if

- Even in the finite volume, the superselection rule distinguishes them:
  $\langle\Phi_1|O(x)|\Phi_2\rangle = 0$ for any local observable $O(x)$ and any finite volume.
- There is no dynamical domain wall connecting those two states, Φ_1 and Φ_2.

Based on this new selection rule, $|\chi = 2\pi/p\rangle$ and $|\chi = 0\rangle$ are distinguished by universes.

Summary

- In YM, we usually sum up all instanton sectors for locality.
- Still, we can modify the instanton sum without changing local dynamics. Locality of QFT is still satisfied.
- By modified instanton sums, the global nature of dynamics is affected:
  - $\theta_{\mathrm{YM}} \sim \theta_{\mathrm{YM}} + 2\pi/p$.
  - $|\chi = 2\pi\ell/p\rangle$ with different $\ell = 0, 1, \dots, p-1$ are distinguished as universes.